Math 107-Sec 450 **Exam 2** **Name:** ______________________

Spring 2001 **Recitation Instructor:** ______________________

No.	1	2	3	4	5	6	7	Total
score								

1. (24 points, 8 pts. each) A fish tank has a square base whose length is 6 feet and rectangular sides of height 2 feet. Assume that the tank is filled with water weighing $\rho = 62.5\ lb/ft^3$.

   a. Find a Riemann sum whose value approximates the work required to pump all of the water over the top of the tank.

   b. Write down **but do not evaluate** an integral whose value is exactly the work required to pump all of the water over the top of the tank.

   c. Write down **but do not evaluate** an integral whose value is exactly the force exerted by the water on one side of the tank.

2. (16 pts, 8 pts each) A rod of length 2 meters lying on the x-axis on the interval $[0, 2]$. Assume its density x meters from the origin is given by $\rho(x) = 5 + 4\cos(2x)$ kg/m.

   a. Find a Riemann sum whose value approximates the mass of the rod.

   b. Write down **but do not evaluate** an integral whose value is exactly the mass of the rod.

3. (12 pts) Find the first 3 non-zero terms of the Taylor series of the function $f(x) = \ln(1 - 3x)$, about $x = 0$.

4. (10 pts) Given that

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + ..., \quad -\infty < x < \infty. \tag{1}$$

By using (1) only, find the Taylor series of $f(x) = x^3 e^{-x^2}$ about $x = 0$.

5. (12 pts) Consider the power series

$$\sum_{k=0}^{\infty} \frac{k}{4^k} x^k = \frac{1}{4}x + \frac{2}{4^2}x^2 + \frac{3}{4^3}x^3 + ... \tag{2}$$

Use the ratio test to determine the values of x for which (2) is convergent. **Do not discuss the convergence of the series at the end points of the interval of convergence**.

6. (12 pts) Find the exact value of the series

$$20 + 3(\frac{2}{5}) - 3(\frac{2}{5})^2 + 3(\frac{2}{5})^3 - 3(\frac{2}{5})^4 + 3(\frac{2}{5})^5 - ... \tag{3}$$

7. (14 pts) Suppose that a function $f(x)$ has the following Taylor series about $x = 0$:

$$f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{3^k k^2}{k!} x^{4k+1}, \quad -\infty < x < \infty. \tag{4}$$

a.(8 pts.) Find the exact values of $f^{(101)}(0)$ and $f^{(102)}(0)$.

b.(6 pts.) Find the Taylor series of $f'(x)$ about $x = 0$.